



Munich Personal RePEc Archive

# Employment Effects of Minimum Wages in Inflexible Labor Markets

Orgul Ozturk

University of South Carolina

2006

Online at <http://mpra.ub.uni-muenchen.de/8016/>

MPRA Paper No. 8016, posted 31. March 2008 18:36 UTC

## EMPLOYMENT EFFECTS OF MINIMUM WAGES IN INFLEXIBLE LABOR MARKETS

ORGUL DEMET OZTURK<sup>\*†</sup>

### Abstract

This paper structurally models and estimates the employment effects of minimum wages in inflexible labor markets with fixed employment costs. When there are fixed costs associated with employment, minimum wage regulation not only results in a reduction in employment among low productivity workers but also shifts the distribution of hours for the available jobs in the market, resulting in scarcity of part-time jobs. Thus, for sufficiently high employment costs, a minimum wage makes it less likely for "marginal" workers to enter and stay in the labor market and has important employment effects. I estimate the model using survey data from Turkey. I find significant reduction in employment due to the loss of part time jobs caused by the national minimum wage policy in this highly inflexible labor market.

Keywords: Fixed employment costs, labor market inflexibility, minimum wage, female labor force participation, part-time jobs, hours constraints

(JEL: J2 J3 E2)

---

<sup>\*</sup>University of South Carolina, Economics Department, Moore School of Business, Columbia SC, 29208. e-mail: [odozturk@moore.sc.edu](mailto:odozturk@moore.sc.edu)

<sup>†</sup>I am grateful for valuable feedbacks provided by John Kennan, James Walker, Maurizio Mazzocco, John Gordanier, Insan Tunali, Seokjin Woo, Sudip Gupta, Jeremy Sandford, Shiv Saini, Jonathan Hore, Hugette Sun, Lucas Davis and an anonymous referee. I also thank the participants of International METU Economics Conference, Annual MEA Conference, University of Wisconsin-Madison Labor Workshop and workshop attendees at Concordia University in Montreal, Kansas State University, Lake Forest College, University of South Carolina, and University of South Florida and the participants of 2007 ZEW workshop "Institutions and Labor Markets".

## I. INTRODUCTION

Most research on minimum wages concentrates on the labor market movements on the margin of employment. Analyses are done to either recover the ratio of workers losing their jobs when the minimum wage increases or the ratio of individuals who are motivated by higher wages to get a job. This paper provides a framework that makes it possible to look at the effects of minimum wages on a variety of labor market outcomes, including but not limited to employment. It shows that having a minimum wage in an inflexible labor market can potentially eliminate jobs with flexible short workweeks, or part-time jobs. This not only reduces employment but also increases the average hours worked in the labor market. This paper also provides a framework which can incorporate minimum wage non-compliance, and informal job markets as alternatives to formal markets as a source of flexibility for marginal workers.

The data used in this paper comes from Turkey, where the minimum wage is binding for a significant portion of workers and there is little or no wiggle room for employers to mitigate the effects of the minimum wage. In many developing countries, the level of the minimum wage is set as a living wage for a family, not for an individual, as the main target group is the male breadwinner. Thus, the employment effects would be expected to have different dimensions than what is addressed by the existing literature.

This paper isolates the minimum wage, and studies its effect on individuals' participation decisions when combined with other labor market "inflexibilities" using micro data. The effect of institutional inflexibilities on employment is studied extensively in the European context, for example, by Bertola (1990) and Blanchard and Jimeno (1995), in attempts to explain high unemployment rates.<sup>1</sup> However, in most of these studies minimum wages are not modeled separately but aggregated in a general measure of labor market flexibility, and employment effects are analyzed on macro data.

The main claim of the paper is that it is prohibitively expensive for firms to employ workers for short workweeks at a minimum wage when the labor market is inflexible due to fixed employment costs. Thus, employers offer contracts that specify a minimum number of hours to be worked. This results in a shift in the distribution of hours for the available jobs in the market, restricting the number of part-time jobs. Part-time jobs play a crucial role in participation decisions of marginal workers, especially women, since women may prefer flexibility with regard to hours of work. Part-time jobs in many cases serve as a gateway to full-time jobs and ease the transition from household production to market work. Thus, for sufficiently high employment costs, a minimum wage makes it less likely for the marginal workers to stay in the labor market.

---

<sup>1</sup>Please see Blanchard (2005) for a detailed review of the history of unemployment in Europe and the literature it inspired.

Over the last fifty years, participation rates among Turkish women declined significantly. They have stayed unexpectedly low over the last couple of decades, especially in urban areas. This pattern is inconsistent with the general worldwide trend of female labor force participation and social and demographic improvements in Turkey. This paper proposes that Turkish women have low participation rates due to the extreme scarcity of part-time jobs, resulting from the constraints on hours implied by the interaction of the minimum wage and market inflexibilities. While the share of part-time employment among females averages around twenty-five percent in OECD countries, in Turkey only 3.5 percent of employed women hold part-time positions. This paper shows that, indeed, if there were fewer restrictions on work hours, the Turkish female labor force participation rate would have been about six times higher.

Card and Krueger (1995) look at the proportion of workers that were full-time in their New Jersey-Pennsylvania study and they document an increase in full time job incidence in New Jersey (where the minimum wage increase happens). However, to my knowledge there is no other study which looks at the relationship between part-time jobs and the minimum wage.

The paper is organized as follows. The next section introduces theoretical model. The third section gives the econometric specification of the model. Section 4 provides background on female labor force participation behavior in Turkey. Section 5 explains the details of estimation, and reports the estimation results. Section 6 provides counterfactual simulations and discusses policy implications. Section 7 concludes the paper.

## II. MODEL

The model used in this paper builds on the labor supply model introduced in Moffitt (1982). In this model each worker faces a restriction on the lowest number of hours she can work: her required minimum hours. She also chooses her desired number of hours, which she can work only if her desired workweek is longer than her required minimum. If her desired workweek is shorter than her required minimum she is considered to be "constrained" and has to choose between working more hours than she wants or not working.

I extend this base model by modeling the marginal productivity determination and letting wages vary by the length of workweek. Since, the per hour fixed cost of employment decreases as the workweek gets longer, average productivity gets higher. Thus, employers are willing to pay higher hourly wages for longer workweeks, which generates a full-time wage premium within the model. The addition of increasing average productivities and the zero profit condition to the model leads to a different modeling of the constraints on working hours.

In Moffitt's model, if the difference between the required minimum and the desired length of workweek is greater than some estimated level,  $D$ , the worker chooses not to work when constrained. This  $D$  is a function of the shape of the individuals' indifference curves, but is treated as constant

across workers in Moffitt's model. In my model, instead of estimating such a constant, I allow workers to make utility comparisons when constrained, and choose the utility maximizing option from this constrained set. Thus, I allow  $D$  to vary across individuals.

I will introduce the model in two subsections: the first subsection analyses how the interaction of the minimum wage and fixed costs results in constraints on hours. The second subsection explains how supply side decisions are affected by these constraints. Table 1 summarizes the notation used.

Table 1: Notation

$\pi$	: Marginal productivity of the potential worker
$f$	: Fixed cost of employment per week per employee (dollars)
$w_h$	: Hourly wage = average productivity $\left( w_h = \frac{\pi h - f}{h} < \pi \right)$
$w^{min}$	: Minimum hourly wage
$h^*$	: Desired hours (length of workweek maximizing potential worker's utility)
$L^*$	: Optimal level of leisure ( $h^* + L^* = T = \text{weekly time endowment}$ )
$h_0$	: Required minimum hours
	$\left( \begin{array}{l} \text{Hours of work required for a worker to produce the value of} \\ \text{the minimum wage on average per hour. Required minimum hours is the} \\ h \text{ that solves } w^{min} = \frac{\pi h - f}{h} \text{ that is } h_0 = \frac{f}{\pi_i - w^{min}} \end{array} \right)$
$h_{00}$	: Absolute required minimum hours
	$\left( = \frac{f}{\pi} \right)$ . This is also the required minimum hours when there is no minimum wage

### A. Demand Side-Sources of the Constraints on Hours of Work

Consider an economy where technology is linear and labor is the only input of production. Each potential worker has a constant marginal productivity ( $\pi$ ). Given such a technology, firms will offer everyone jobs with working hours they optimally choose to supply ( $h^*$ ) at an hourly wage ( $w_h$ ) equal to their average productivity, which is equal to their marginal productivity ( $w_h = \pi$ ).

Now, consider two individuals with different marginal productivities ( $\pi_a > \pi_b$ ) but the same level of desired hours. Any given firm will hire them both and pay hourly wages ( $w_h$ ) equal to their average productivities,  $w_{ha} = \pi_a$  and  $w_{hb} = \pi_b$  respectively. However, if there is a minimum wage in this economy (suppose it is set at a level between  $w_{hb}$  and  $w_{ha}$ ), no worker with an average productivity less than the minimum wage ( $\pi_b = w_{hb} < w^{min}$ ) will be offered any job. Since average productivities are constant, there will be no constraints on the hours worked by the individuals who are offered jobs. That is, a worker with productivity  $\pi_a$  can still work her desired level of hours. Nevertheless, an individual with a productivity  $\pi_b$  will no longer be employed by anybody.

Suppose now there are costs associated with each job equal to  $f$  dollars per worker for each workweek. As a result, each worker starts producing a surplus value for the employer after the first

$\frac{f}{\pi}$  hours. I call this "the absolute required minimum hours" and denote it by  $h_{00}$ . The key point in this model is the increasing average productivity: the cost of employment will make a worker's average productivity, as well as the hourly wage she earns, dependent on the number of hours she works. This hourly wage is less than what it is when there are no fixed costs since now the total value of the workers production will be reduced by the costs associated with her employment <sup>2</sup>. The average hours curve in Figure 1 shows us the "menu" of jobs (defined as a pair of working hours and an hourly wage) the worker will consider in her utility maximization.

However, minimum wage regulation is such that it does not take the existence of fixed costs into account and requires a constant hourly wage independent of the length of the workweek ( $w^{min}$ )<sup>3</sup>. Therefore, when there are fixed costs, minimum wage regulation creates an interval of hours where the average productivity is lower than the minimum wage for every worker. This results in restrictions on the minimum number of hours each worker can work ( $h_0$ ). That is, some of the jobs (jobs with hours less than  $h_0$ ) in the "menu" the worker considers will no longer be offered to her by the employers. Solving the hourly wage equation for the  $h$  where average productivity is equal to the minimum wage gives

$$h_{0i} = \frac{f}{\pi_i - w^{min}} \quad (1)$$

which is an increasing function of the fixed employment cost and the minimum wage, but a decreasing function of the worker's productivity. Figure 1 illustrates how the minimum number of hours that a certain worker needs to supply decreases as the productivity level increases ( $\pi_a > \pi_c > w^{min} \implies h_{0a} < h_{0c}$ ). In other words, worker with a higher productivity will have more options on her "menu of jobs" with a wider range of hours.

---

<sup>2</sup> $w_h = \frac{\pi h - f}{h} < \pi$

<sup>3</sup>In this paper, minimum wage is set to be an hourly wage for two reasons. First, most of the relevant literature works with data where the minimum wage is hourly. Second, the employment effects implied by this model will be in the same direction but magnified if the minimum wage was set in any other manner, like as a weekly or a monthly minimum wage.

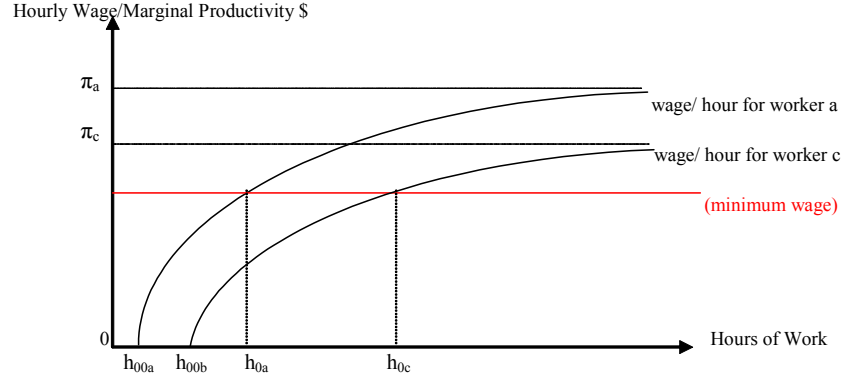


Figure 1

### B. Supply Side-Participation Decision with Constraints on Working Hours

Suppose that on the supply side of the labor market, there are individuals maximizing the utility function  $U = U(C_i, h_i ; A_i, \epsilon_{1i}, \epsilon_{2i})$  choosing the amount of work hours ( $h_i^*$ ) they want to supply and the level of a composite market good ( $C_i^*$ ) they want to consume given their individual observable characteristics ( $A_i$ ) and the unobservable heterogeneity in terms of hours preference and productivity ( $\epsilon_{1i}, \epsilon_{2i}$ ).

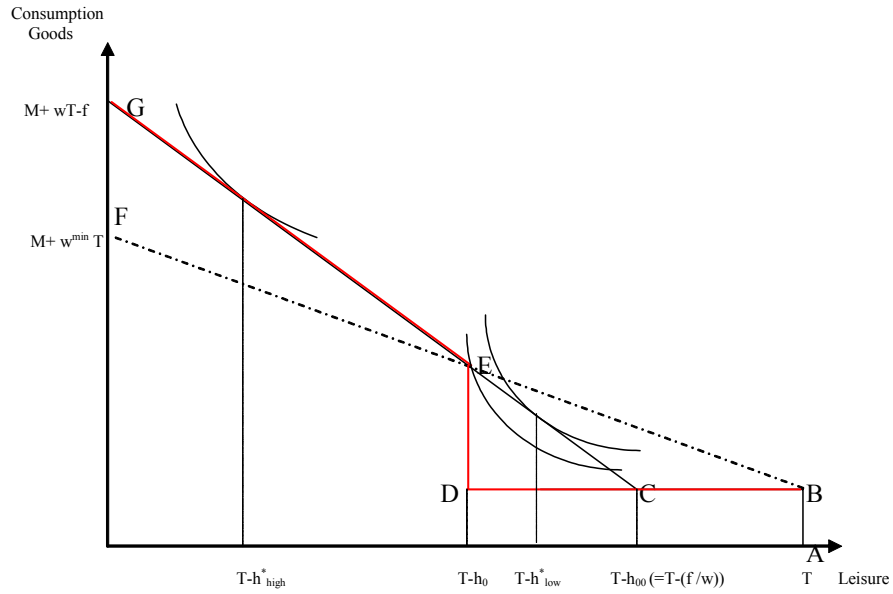


Figure 2

If the potential worker wants to supply a higher number of hours than she is required as a minimum, she will not be restricted. However, even a worker with productivity higher than the minimum wage will face unemployment if she has a low taste for work (or higher opportunity cost of working). Figure 2 demonstrates this situation, showing two workers with the same productivity— $\pi$  (slope of the line  $CEG$ ) which is higher than the minimum wage (slope of the line  $BEF$ )— but different levels of desired hours  $h_{high}^*$  and  $h_{low}^*$ . An individual with desired hours equal to  $h_{high}^*$  will not be constrained by the demand side. However, an individual with  $h_{low}^*$  will face the choice between working  $h_0$  (corresponding to the corner labeled  $E$ ) and not working at all (the corner labeled  $B$ ) since she will not be offered her optimal job any more.

The above discussion shows that a minimum wage can have significant employment reducing effects when there are high fixed costs. Moreover, these effects are felt more severely by low-productivity individuals, since lower productivity implies a higher required minimum hours. Individuals with a low taste for market work (or high opportunity cost) who supply less hours to the market will also be affected more by the minimum wage in this market.

### III. ECONOMETRIC SPECIFICATION

In the model, there are workers who work their desired hours and workers who work their required minimums. Moreover, there are three groups of non-workers. The first group consists of the ones who willingly opt out of the labor market regardless of the minimum wage. The second group includes the ones who choose not to work the long hours they are offered. The third group is the group of non-workers who wants to work, but are not offered any jobs because their marginal productivity is less than the minimum wage. The main econometric difficulty arises from the fact that it is not possible to observe which workers are at their required lower bounds and which are working their desired hours. Moreover, I cannot observe which non-participants are constrained and would like to supply positive hours and which would not. I only know who is working and who is not, and the actual working hours for each worker. I assume the behavioral structure producing the observed behavior and utilize the model to recover the parameters that maximize its fit. I start by assuming that everybody has the following utility function,

$$U(C_i, L_i; A_i, \epsilon_i) = \left( \frac{\alpha_2(T - L_i) - \alpha_1}{\alpha_2^2} \right) \exp \left( \frac{\alpha_2(\alpha_0 + \alpha_2 C_i + \alpha_3 A_i + \epsilon_{1i}) - \alpha_1}{\alpha_2 h_i - \alpha_1} \right)$$

which is maximized subject to the following set of constraints<sup>4</sup>

---

<sup>4</sup> $C_i = M_i + w_{ih} h_i \implies C_i = M_i + \left( \frac{\pi_i h_i - \gamma_i f}{h_i} \right) h_i \implies C_i = M_i + \pi_i h_i - \gamma_i f.$



$$C_i = M_i + \pi_i h_i - \gamma_i f$$

$$L_i + h_i = T$$

where  $A_i$  is a vector of demographic characteristics,  $M_i$  is non-labor income,  $C_i$  is the composite good (the numeraire),  $L_i$  is leisure and  $T$  is the fixed weekly time endowment that can be divided between leisure and work.  $\gamma_i$  is a dummy which is equal to 1 if the individual works and 0 if not. This "weird" utility function is chosen because it gives a linear labor supply function which is widely used in the literature. That is, conditional on choosing to work a positive number of hours, the optimal number of working hours is given by the following expression:

$$h_i^* = T - L_i^* = \alpha_0 + \alpha_1 \pi_i + \alpha_2 (M_i - f) + \alpha_3 A_i + \epsilon_{i1} \quad (2)$$

Restrictions  $\alpha_1 > \alpha_2 h_i^*$  and  $\alpha_2 \leq 0$  guarantee quasiconcavity of the utility function and its monotonicity in disposable income. While  $\alpha_1 > \alpha_2 h_i^*$  implies that the compensated wage effect is non-negative, the uncompensated wage effect,  $\alpha_1$ , can be positive or negative.<sup>5</sup> The second constraint  $\alpha_2 \leq 0$  assures that leisure is not inferior.

Marginal productivity is given by the following equation

$$\pi_i = \exp(X_i \beta + \epsilon_{i2}) \quad (3)$$

where  $X_i$  represents individual productivity characteristics. The error terms  $\epsilon_1$  and  $\epsilon_2$  are assumed to be independently distributed as normals with means equal to zero and standard deviations equal to  $\sigma_1$  and  $\sigma_2$  respectively.

If an individual desires to work a positive number of hours, has marginal productivity greater than the minimum wage, and has higher utility from working her required minimum hours than not working, she actively participates. Otherwise she does not work. As stated earlier, I do not observe either  $h^*$  or  $h_0$ . However, if the individual is active in the labor market, I know  $h_i$ , the observed working hours. Since, in this model,  $h_i$  is either desired hours or minimum required hours, I can use the conditions governing the participation decision, to construct the rules determining the choice of work hours. Figure 3 illustrates the regions regarding participation behavior in the plane of "desired"

---

This budget constraint can also be used to model fixed costs that are directly incurred by the employees facing a single wage offer ( $\pi$ ) and choosing how much to work at the hourly wage net of the fixed costs ( $w_h$ ). With the production technology used in this paper, these two models will give identical results with the same interpretation.

<sup>5</sup>See Hausman(1980) or Pencavel (1986)

and “required minimum” hours.<sup>6</sup>

As long as the individual desires longer workweeks than the minimum workweek that she is offered, she is not constrained by the minimum hours requirement and she works her desired hours. However, when the desired length of her workweek, given that it is more than zero, is shorter than the minimum offered to her, she is forced to choose between not working and working the required minimum. She works  $h_0$  hours at minimum wage<sup>7</sup> only if it is more desirable to do so than not working. That is,

$$h_i = h_i^* \quad \text{if } h_i^* > h_0 \text{ and } \pi_i > w^{\min} \quad (I)^8$$

$$= h_0 \quad \text{if } h_0 > h_i^* \text{ and } U(h_i = h_0) > U(h_i = 0) \quad (II)$$

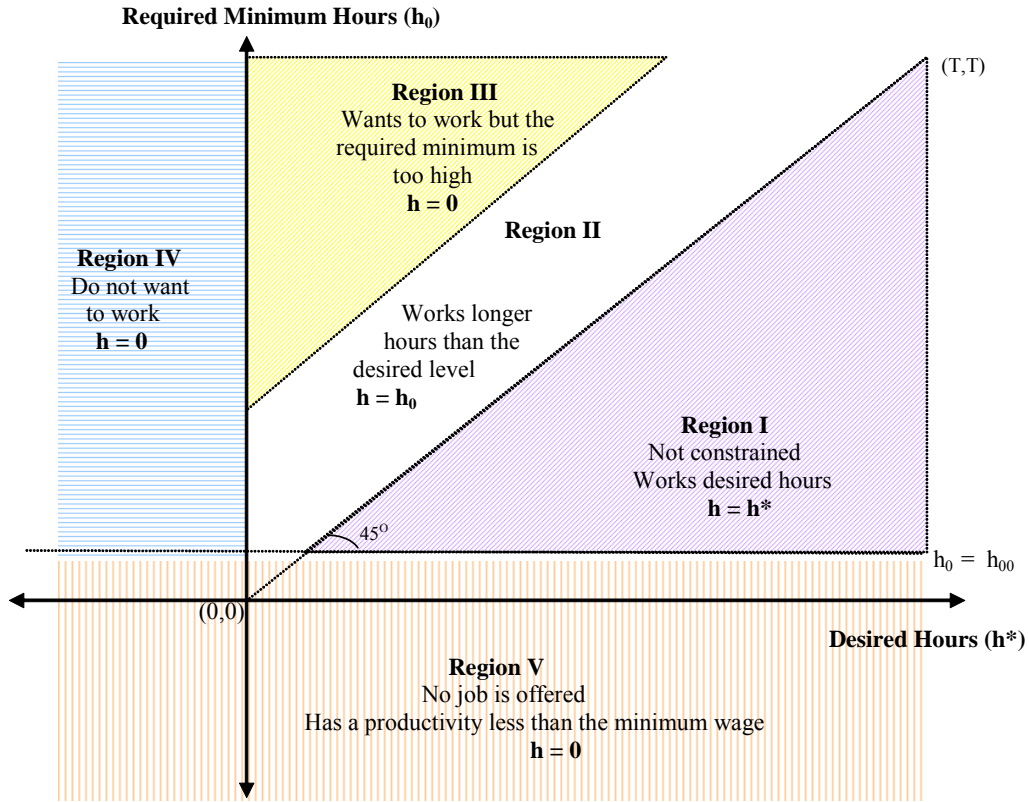


Figure 3

<sup>6</sup>See the first section in the appendix for an illustration of the hours choice using the utility function.

<sup>7</sup>Minimum wage is equal to the (minimum) hourly wage at the required minimum hours.

<sup>8</sup>This constraint means that the worker will only be offered a job with a positive wage if her productivity is greater than the minimum wage. This constraint is imposed for technical reason during the optimization since if  $\pi_i < w^{\min}$  then  $h_0 < 0$  and is less than  $h^*$ . By imposing this constraint, I can substitute minimum wage as a wage for the job that comes with minimum required hours since minimum wage is equal to the (minimum) hourly wage at the required minimum hours.

Similarly, there are three groups among the non-participants. The first group is the group of individuals who would supply positive hours if they were not constrained. They are asked to work longer hours than they are willing to supply. When facing this set of choices, they prefer not to participate. On the other hand, for the second group of non-participants, the desired workweek is less than or equal to zero. They are the ones who willingly choose not to participate. The last group of non-workers consists of individuals who are undesirable in the market when there is a minimum wage, that is, their productivity is lower than the minimum wage. In summary,

$$\begin{aligned}
h_i = 0 & \quad \text{if } h_0 > h_i^* > 0 \text{ but } U(h_i = h_0) < U(h_i = 0) & (III) \\
& \text{or} \quad \text{if } \pi_i > w^{min} \text{ but } h_i^* \leq 0 & (IV) \\
& \text{or} \quad \text{if } \pi_i < w^{min} & (V)
\end{aligned}$$

Given these regions of participation, the probability of working  $h_i = h$  hours can be written as probability of observing  $h$  either as  $h^*$  or as  $h_0$ , that is

$$Q = \left( \begin{aligned} & \Pr(h_i = h^*, h_i^* > h_0 > 0 | X_i, A_i, \sigma_1, \sigma_2, w^{min}, M_i) \\ & + \Pr \left[ \begin{aligned} & h = h_0, h_0 > h_i^* > 0, U(h_i = h_0) \\ & > U(h_i = 0) | X_i, A_i, \sigma_1, \sigma_2, w^{min}, M_i \end{aligned} \right] \end{aligned} \right)$$

The probability of not working, on the other hand, is the combined probability of being in regions *III*, *IV* or *V* and can be formulized as

$$q = \left( \begin{aligned} & \Pr(h_0 > h_i^* > 0, U(h_i = h_0) < U(h_i = 0) | X_i, A_i, \sigma_1, \sigma_2, w^{min}, M_i) \\ & + \Pr(h_i^* = 0, \pi_i > w^{min} | X_i, A_i, \sigma_1, \sigma_2, w^{min}, M_i) \\ & + \Pr(w^{min} > \pi_i | X_i, A_i, \sigma_1, \sigma_2, w^{min}, M_i) \end{aligned} \right)$$

Thus<sup>9</sup>, the log likelihood function,  $\log L$ , is

$$\log L = \sum_{h>0} \log Q + \sum_{h=0} \log q$$

I estimate the model using Maximum Simulated Likelihood (MSL). This method replaces the actual probabilities defining the likelihood function with simulated ones. The simulated probabilities are generated by a Logit-Smoothed Accept-Reject Simulator (LS-AR Simulator).

#### IV. FEMALE LABOR FORCE PARTICIPATION IN TURKEY

Despite the demographic and social changes over the last 50 years, the female labor force participation rate has decreased significantly in Turkey over this period. The female labor force participation rate was 72 percent in 1955 but declined to 23 percent in February 2005. In 2005, the participation rate was only 18 percent among urban women (SIS HLFS, 2005). Over this same period, participation rates of women on average doubled worldwide, and almost tripled for married women in most countries going through similar social changes.

The initial drop in the participation rate has been attributed to the massive urbanization of the workforce after the 1950s. Small scale, family-level agriculture had been employing nearly all of the women in rural areas. Since the distinction between household duties and work is blurred in agriculture, it is easier for rural women to meet the conditions to be considered as employed. It has been argued that when women move to the cities, they cannot find a place for themselves in the labor force of urban Turkey (Dayioglu, 1998; Ozar, 1996; Tunalı, 1997). In cities, market work and household duties are incompatible. Hence, women have to concentrate on one of them. Most of these women have little human capital, so they are employed in “marginal” jobs. Faced with this, most choose not to participate in the workforce. However, the continuing decline in the participation rate is unexpected since the social status of women has improved significantly over these years.

There is no study yet that provides a convincing explanation why the Turkish economy is incapable of utilizing the increasing productivity of women. The model developed in this paper is estimated with Turkish female labor force data and explains the low participation rates among urban females through constraints on hours in the job market, i.e. the lack of part-time jobs.

This is not the first paper that calls attention to the link between the lack of part-time jobs and the low female labor force participation rate in Turkey (Baslevent, 2001). However, there is

---

<sup>9</sup>See the second section in the appendix for details of this derivation.

no explanation for the scarcity of part-time jobs. The stylized model of a labor market analyzed in this paper captures the fundamental characteristics of the Turkish labor market. According to various OECD reports, Turkey is among the least flexible labor markets worldwide with regards to employment. The main source of inflexibilities in this market are the policies regarding non-wage monetary burdens associated with employment implied by the labor law which was in effect between 1947 and 2003, roughly the time period we are interested in. The absence of a linear relationship between tax and benefit payments, and hours of work (Tunali, 2005) makes part-time employees very undesirable in the Turkish labor market.

Women, while looking for a job, may prefer flexibility with regard to hours over pay. For example, Falzone(2001) shows with US data that part-time work offers an efficient alternative for married women in the labor market when earnings are not the only consideration. In most countries, part-time positions are observed as low-pay, low-benefit jobs frequently occupied by women. However, in Turkey, existing part-time jobs exhibit different characteristics as illustrated in Table 2. In the Turkish labor market, part-time workers earn on average almost three times as much as full-time workers. Tables 3 and 4 help clarify this picture. Most part-time workers are university graduates; high productivity workers. The share of part-time workers is 31 percent among college graduate women in my data. Among women with less education, on the other hand, this ratio is only 10 percent. The summary statistics also show that the higher the years of schooling completed, the lower the average number of hours worked per week. The model in this paper nicely fits these observations and provides an explanation for the existence of "part-time wage premium" in the data.

Table 2: Wages - Part-time vs. Full-time Jobs

	# of obs.	mean	st.dev.	min.	max.	median
if h < 40	87	1.85	5.29	0.11	48.94	1.17
if h ≥ 40	474	0.74	0.97	0.05	18.43	0.58

Table 3: Share of Part-timers and Education

	# of obs.	% part-timers
college graduates	154	31.13
non-college graduates	407	10.03

Table 4: Hours of Work and Schooling

	# of obs.	mean	st.dev.	min.	max.	median
primary school or less	189	42.79	9.03	15	84	40
middle school	35	42.48	4.68	40	58	40
high school	183	40.36	5.98	20	64	40
college	154	35.58	9.19	15	54	40

This observation surprises many scholars and some even claim that there is a wage premium to part-time jobs in Turkey. This interesting phenomenon can be explained with the model introduced in this paper; average part time wages are higher simply because there are almost no part-time jobs among the low paying jobs in the market.

## V. ESTIMATION

### A. Data

The data set used is from the Turkish Household Labor Force Supply Survey. This survey is conducted biannually by the State Institute of Statistics of Turkey from 1988 to 1999, and quarterly since 2000. In total, 14, 000 to 23, 000 households are surveyed each time, both from rural and urban areas. The analysis here uses the data from the October 1988 round of this survey.

In the 1988 round, 102, 062 individuals residing in 22, 320 households nationwide are surveyed. In this data set, participation for women is around 18 percent in cities, very similar to the census results. Participation rates vary greatly with education and marital status. There are significant drops in participation rates as education falls below college level (73 percent at the college level and 8 percent for primary school graduates) and as women get married (38 percent for singles, 11 percent for married). In the survey, nonworking women are asked if they would like to work and the ratio of those who are ready to start working is higher among married and low-educated (although slightly in some cases) suggesting that more of those women are the ones who are staying out of the market.

For the empirical analysis, I use a sub-sample of 6, 445 women between the ages of 20 and 55 who are married and living together with their husband in cities with 400, 000 or more people. Women in the sample either did not work the week preceding the interview or they were employed as wage and salary workers. I use data only on women who are working at most one job and who are not currently enrolled in school, either full-time or part-time. Table 5 gives the descriptive statistics for the women in my sample.

Table 5: Descriptive Statistics

Variables	# of obs.	mean	st.dev.	min.	max.	median
Hours worked (if working)	561	40.01	9.16	15	84	40
# of children of ages 0-5 (conditional on having a child)	2804	1.38	0.61	1	4	1
# of children of ages 6-14 (conditional on having a child)	3753	1.86	0.94	1	6	2
Education	6445	4.66	3.671	0	15	5
Age	6445	34.62	9.16	20	55	34

In this sub-sample, the mean education is about five years. Seventy-four percent of the women interviewed have seven or less years of schooling (last degree they have completed is primary school). University graduates constitute six percent of the women and about thirty-seven percent of the workers in the sub-sample. The labor force participation rate for this sub-sample is about nine percent. These women work forty hours on average. Eighty-three percent of working women work forty hours or more and only five percent work twenty hours or less (Eight percent of women work between twenty-five and forty hours, nine percent work less than twenty-five hours).

Table 6: Variable Definitions

$A_i$	demographic variables	
	age	between 20-55
	squared age	age squared/100
	years of schooling	0 = no schooling 3 = literate but has no degree 5 = primary school 8 = middle school 11 = high school 15 = college or more
	squared years of schooling	years of schooling squared/100
	young children	number of children between ages 0-5
	squared young children	number of young children squared
	older children	number of children between ages 6-14
	squared older children	number of older children squared
$X_i$	productivity variables	
	middle school	dummy (0-1)
	high school	dummy (0-1)
	college	dummy (0-1)
	potential experience	age - years of schooling - 6 (6 is the age at schooling begins)
	squared potential experience	potential experience squared/100
$M_i$	non-labor income	household income-own labor income number of household members

I use different educational indicators, family variables and individual demographic indicators as the explanatory variables in the estimation. Table 6 lists the variables used in all steps of estimation with explanations. There are a few problems with the data; for example, wages and a non-labor income measure are not directly available. There is also no record of asset income. I use the weekly value of per member income of the household excluding women's own earnings as a proxy for the non-labor income. I only have monthly incomes recorded, thus I divide the figures by four to get

an approximate weekly number. In the survey, individuals are asked their usual working hours per week, and how much they worked last week. However, they report how much they earned in the month preceding the interview. I approximate the weekly labor income using these figures, making sure that the individuals were working for the whole month for which they report the income. Three observations which are not meeting this criterion are excluded from the sample used for the analysis

The data set is cross-sectional and the nominal level of minimum wage is constant across the country. I generate variations in the minimum wage using the province level CPI<sup>10</sup>. I keep the prices in Ankara (the capital city) as the base and divide the minimum wage in the other provinces with ratio of their prices to the prices in the capital. This measure reflects the differences in the real value of minimum wage across individuals even though they all face the same nominal level. I made the same adjustment to non-labor income and wage measures. I convert all values into US Dollars using average Dollar/Turkish Lira exchange rate for October 1988, the month that the survey took place.

## **B. Results**

Married women have higher-valued outside options since the division of labor in the household requires them to be the main producers at home in most cultures. Thus, females of a given market productivity are expected to supply fewer hours of labor than their male counterparts. These women are also expected to make the non-participation decision more easily if they are forced to work long hours. This is what we observe in the data. The share of housewives among non-participating women is strikingly high in Turkish data; Seventy-nine percent of women who do not participate in the labor force stated being a housewife as the reason. Household duties keep women at home when the labor market options are not attractive enough. My estimates provide support to this not-so-new idea. Looking at the Table 7 we can see that having young kids in the household decreases the desired workweek. While having only one young child at the household reduces the desired hours by little less than six hours, having two young children reduces desired hours by more than ten. The effect of having older kids is similar on hours choice but its magnitude diminishes as the number of children in this age group increases in the household. A woman who has a child between ages six and fourteen wants to work about three hours less compared to her "twin" with no children of ages six to fourteen.

The estimates of the marginal productivity parameters suggest significant economic returns to education especially at the college and high school level. Everything else equal, college graduate women earn about hundred percent more per hour compared to women with no education. The wage return to a college education is more than double the wage return to a high school degree compared to the women with no education. This partially explains the discrepancy between participation rates across different education levels.

---

<sup>10</sup>I use 1995 prices, the earliest year for which CPI exists for all the provinces I have in the data.



The mean of the productivity estimates is fifty-four cents for the working individuals. That is, the average worker produces fifty-four cents worth of goods or services per hour. The distribution of these productivity measures has a standard deviation of seventeen cents, with values ranging between three cents and two dollar and thirty-four cents for the entire sample. According to these estimates, nine percent of the women have simulated productivities that are less than the minimum wage level, which ranges between thirty-two and thirty-four cents across fourteen cities.

The number of desired work hours decreases in non-labor income, but the effect is not very significant economically. In this case, non-labor income is approximated using the labor incomes of the other family members. The sum of family income excluding the wife's income is divided by the family size. Keeping this in mind, the estimate for  $\alpha_2$  suggests that by every hundred extra dollars the other family members earn per person the desired hours of a potential worker decreases by three hours per week.

Table 7: MSL Estimates

Desired and Required Minimum Hours	estimate	st.dev
constant ( $\alpha_0$ )	21.49	3.82
wage ( $\alpha_1$ )	4.81	1.01
non-labor income ( $\alpha_2$ )	-0.03	1.08E-03
years of schooling	0.88	0.09
squared years of schooling	-3.55	0.65
age	0.84	0.17
squared age	-1.67	0.25
young kids	-2.47	0.77
older kids	-3.49	0.42
squared young kids	-2.71	0.59
squared older kids	0.52	0.17
fixed employment cost (f)	5.38	0.26
Marginal Product		
constant	-1.54	0.01
middle school	0.20	0.01
high school	0.44	0.02
college	0.93	0.03
potential experience	1.6E-03	3.03E-04
squared potential experience	-5.4E-05	5.23E-06
$\sigma_1$	8.11	0.09
$\sigma_2$	0.42	0.01
log likelihood	-3028	

The coefficients on age variables imply that desired hours increases by age up to age thirty-three and declines thereafter. Such a pattern in terms of hours worked does not appear in the data. However,

we know that not all workers work their desired hours; according to the simulations about forty percent of the workers are constrained to work at their required minimum. Given this, Figure 4 illustrates why we fail to observe such a pattern with hours data. The hours worked at the low and high end of the age distribution is still high due to the higher proportion of constrained workers in those age groups. In other words, because they have higher desired hours, a smaller proportion of middle-aged workers are constrained.

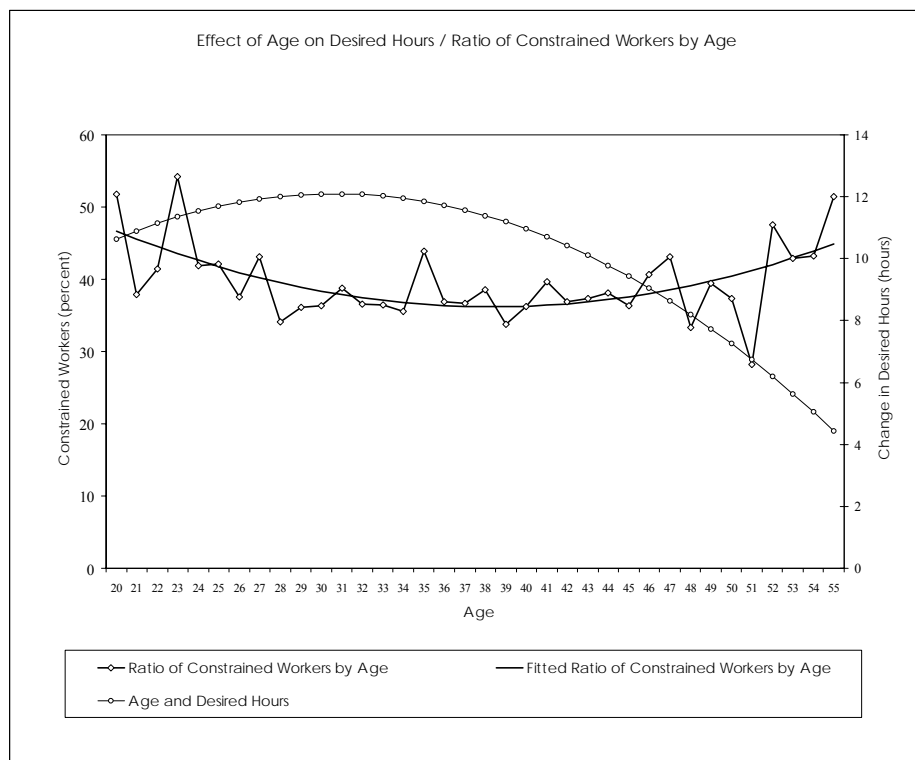


Figure 4

The estimate for  $\alpha_1$  may seem to be small suggesting that a dollar increase in the wage will increase the desired hours by five hours given the range of wage estimates. For the average worker one extra dollar per hour is about a two hundred percent increase in hourly wages. This is in line with the findings of several papers on Turkish female labor market activity. Tunali (1995), for example, finds that the wage elasticity of hours supplied is almost zero among Turkish women.

The fixed employment cost is estimated to be about five dollars and forty cents. As mentioned before, an average worker works forty hours per week and makes about fifty-four cents per hour. In this case, five dollars and forty cents corresponds to about twenty-five percent of the weekly

earnings. About thirty-one percent of all labor costs in Turkey(in 1990) was non-wage payments.<sup>11</sup>

### C. Participation Regions

The estimated participation rate from the model is 8.87 percent. Table 8 summarizes the participation probabilities associated with regions in Figure 3. According to these estimates, eighty percent of all women are restricted in the sense that they want to supply positive hours of work but either are not desired as workers or are constrained by high required minimum hours. Conditional on being a non-participant, about twenty-five percent of women want to work and are welcome in the market, but are asked to work more hours than they are willing to supply. About sixty percent of women are not offered any job.

Table 8: Participation Regions

Event	Definition	Probability
$h > 0$	participation	0.09
$h = h^*$	working desired hours - Region I	0.04
$h = h^{min}$	working required minimum - Region II	0.05
$w^{min} < \pi, h^* > 0, h = 0$	required minimum too high - Region III.	0.25
$w^{min} < \pi, h^* < 0, h = 0$	not want to work - Region IV	0.04
$w^{min} > \pi$	no job offer - Region V	0.62

### D. Fitting the Hours Distribution

Table 9 reports the distribution of the estimated hours. In the simulated data, the average length of the workweek is about forty-one hours. For the women working their required minimum hours, the mean workweek is forty-seven hours long, and for women working their desired hours the mean workweek is thirty-five hours.

Table 9: Distribution of Hours

	mean	st.dev.	min	max
estimated	40.61	13.52	8.54	89.67
$h = h_0$	46.81	15.31	13.64	89.67
$h = h^*$	34.88	8.19	8.54	61.34

<sup>11</sup>TISK(Turkish Employer's Unions Confederation) Website. [www.tisk.org.tr](http://www.tisk.org.tr)

Figure 5 graphs the simulated hours distribution and also shows the distributions for the restricted and unrestricted workers. The relatively high concentration of workweeks around thirty to forty five hours can be considered as a possible explanation for the concentration of the hours distribution around forty hours in the data.

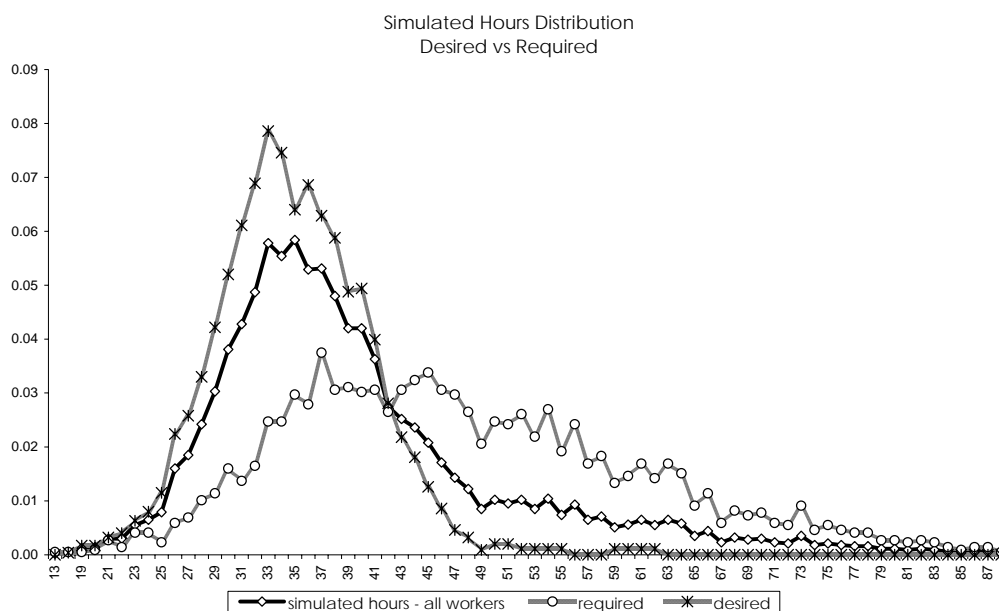


Figure 5

## VI. COUNTERFACTUALS

Given the estimates and the data I can simulate the participation and hours choices under different minimum wage policies. Moreover, I can see how the participation and hours choices could have been with the same minimum wage in a different economic environment, in this case, a labor market with no employment costs. Based on the estimates, I simulate several counterfactual scenarios and analyze transitions across labor market groups under these alternative policies. Table 10 contains the participation probabilities generated via simulations under these counterfactuals.

Table 10: Participation Regions under Counterfactuals

$f=f_{estimate}, w^{min}=0$		
$h>0$	participation	0.48
$w^{min}<\pi, h^*<0, h=0$	not want to work	0.09
$f=0.5*f_{estimate}, w^{min}=w_{data}^{min}$		
$h>0$	participation	0.15
$w^{min}<\pi, h^*>0, h=0$	required minimum too high	0.19
$w^{min}<\pi, h^*<0, h=0$	not want to work	0.04
$w^{min}>\pi$	no job offer	0.62
$f=0.5*f_{estimate}, w^{min}=0$		
$h>0$	participation	0.75
$w^{min}<\pi, h^*<0, h=0$	not want to work	0.09

If the minimum wage is zero (in the presence of the fixed cost), the ratio of women working increases to 48 percent while only 9 percent of women do not want to work.

The fixed cost in this model represents not only technological burdens but also policy-implied costs of employment. Thus, although it is not reasonable to think of an environment without any fixed employment costs, we can think of an environment sans the institutional costs imposed by the regulations, taxes etc. About fifteen percent of women participate when fixed costs are reduced by fifty percent. If there were no constraints in the market, the simulations show that about 60 percent of currently non-working women would obtain jobs. This would increase total female labor force participation to 75, about 9 times the current estimated rate. A simulation without fixed costs indicates that the minimum wage alone explains 42 percent of this total increase. Similarly, a simulation including fixed costs but no minimum wage shows that fixed costs account for only 7 percent of the change. Thus, the interaction of the minimum wage with fixed employment costs accounts for most of the difference.

These changes also affect the distribution of working hours. Table 11 shows that the mean, minimum, and maximum hours worked are lower, indicating more women are working at the low end of hours distribution. This supports the claim that if there were more part time jobs available in the market participation would have been higher.

Table 11: Counterfactual Distribution of Hours

	mean	st.dev.	min	max
$f=f_{estimate}, w^{min}=0$	31.91	7.78	10.28	68.62
$f=0.5*f_{estimate}, w^{min}=w_{data}^{min}$	36.08	12.05	5.99	89.72
$f=0.5*f_{estimate}, w^{min}=0$	28.76	8.56	5.14	68.54

## VII. CONCLUSION

In this paper, I show that the interaction of minimum wages and fixed costs of employment imposes limits on offered working hours and as a result can cause a shortage of part-time jobs. Thus, for sufficiently high employment costs, the institution of a minimum wage affects employment among all workers who prefer flexibility in terms of hours regardless of the productivity level. I estimate the model with Turkish data. My estimates indicate that about 80 percent of all women in Turkey are restricted; they wish to supply positive hours of work, but either have lower than minimum wage productivities and thus are not desired as workers or are constrained by required minimum hours. The key parameter in the model is the fixed cost of employment, which is estimated to be about 5 dollars per week for each employee. The average worker in the sample works forty hours per week and makes about 54 cents per hour. The 5 dollars fixed cost thus corresponds to about 25 percent of weekly earnings. Given that on average 30 percent of all labor costs in Turkey are non-wage expenses<sup>12</sup>, this estimate is a good approximation.

With counterfactual simulations I show that if there were no constraints in the market, total female labor force participation would increase ninefold. About 65 percent of this new participants hold part time jobs. A simulation without fixed costs indicates that the minimum wage alone explains 42 percent of this total increase and a simulation including fixed costs but no minimum wage shows that fixed costs account for only 7 percent of the change. These results support the claim that the impact of the minimum wage is strongest when it is imposed in inflexible market conditions.

There are several assumptions in the model that may be considered restrictive. For example, in the current functional specification of the model, there is no place for a non-monotonic relationship between hours supplied and fixed costs. Moreover, there is no room for non-linear responses to wages. Implications of the model for employment decisions do not change if the technology is modified in order to allow alternative constraints and wage structures. For example, an S-shaped hours-productivity relationship (Barzel, 1973; Moffitt, 1984), which is considered a more realistic approach, would lead to both lower and upper bounds on the length of workweeks acceptable to the employers. This would strengthen the impact of minimum wage on the level of employment even without the fixed costs. In the data, the distribution of hourly wages by workweek is weakly concave, which rejects the idea of a full-time wage premium. I take this as a sign that this model is a reasonable choice for the environment. It implies that part time jobs will be in short supply and high productivity workers will occupy the existing jobs. Low productivity workers will be constrained with higher working hours. Thus, in this environment, the part-time job market may have higher hourly wages on average than the full-time job market. This is quite different from the markets that

---

<sup>12</sup>SIS statistic

are explained with S-shaped budget constraints.

Allowing constraints only on the minimum number of hours workers can work, may also seem limiting. However, an upper limit on working hours does not seem to be an issue in the data. Moreover, I choose not to discretize the choice set of hours, unlike some other studies in the literature, since the main concern is not fitting the distribution of observed hours (mainly the spikes at certain length of workweeks, like 40 hours) but understanding how important these constraints are in explaining the concepts of voluntary and involuntary unemployment. I cannot capture the spikes of observed hours distribution with the estimates. However, model successfully fits the external margins of participation. I also ignore possible heterogeneity of fixed costs due to the lack of variables needed to identify such variation across workers.

This paper offers a stylized model of employment costs. The model restricts the usage of information on employers since this information does not exist for non-workers, thus cannot be used to approximate the latent indices created for each individual. Estimating the model only on workers can improve this aspect of the estimates. However, workers constitute a minority in this data set, which reduces the power of estimation. Thus, the next step is to estimate the model on a data set where employment rates are higher, like the Current Population Survey data. In the mean time, the data set can be enriched by inclusion of single females and maybe males. Married women make non-participation decision easier compared men and single women since they usually have a higher non-labor income to rely on compared to their husbands. It will be interesting to model household as the unit of analysis and estimate the impact that the minimum wages and market inflexibilities have on the intra-household division of labor. Turkish married men work on average 52 hours in my data. This is very high compared to many other countries.

In this model productivity is perfectly observed by employers, and wages are based on productivity. This kind of model of the labor market has been used before to consider minimum wage impacts, dating back to Stigler in the 1940s. It has been criticized before, because it doesn't account for the spike of the wage distribution at the minimum wage (Card and Kreuger, 1995). The model introduced in this paper doesn't suffer from the criticism of Card and Kreuger. One implication of my model is that, while it's based on a Stigler-type view of the labor market, it still ends up with a spike at the minimum, without raising anyone's wages (I see this in the Turkish data - an impressive 48 percent of the blue collar workers are reported to be working at the minimum wage level). This model also can be used as an explanation for high rates of minimum wage noncompliance – if the alternative is being unable to work workers would not complain if they're being paid below the minimum wage when job comes with flexible hours.

## REFERENCES

- Barzel, Yoram, (1973), "The Determination of Daily Hours and Wages", *Quarterly Journal of Economics*, 87(2), pp.220-238
- Bertola, G. (1990) "Job Security, Employment and Wages" *European Economic Review*, June '990, 34(4), pp.851-79"
- Bernstein, J. and J. Schmitt (1998) "Making Work Pay: The Impact of the 1996-1997 Minimum Wage Increase" Washington DC, Economic Policy Institute
- Blanchard, O.J, and Juan Jimeno (1995) "Structural Unemployment: Spain versus Portugal" *American Economic Review*, (2) pp.212-18
- Blanchard, O.J (2005), "European Unemployment: The Evaluation of Facts and Ideas", NBER Working Paper #11750
- Brown, C. Gilroy, C. and Kohen, A. 1982. "The Effect of the Minimum Wage Law on Employment and Unemployment", *Journal of Economic Literature* (20) pp. 487-528.
- Card, D.and A.Krueger (1995), *Myth and Measurement: The New Economics of the Minimum Wage*, Princeton: Princeton University Press.
- Dayioglu, M. (1998), "Labor Market Participation of Women in Turkey" in F. Acar and A. Ayata (eds.), *Women's Identities and Roles in the Course of Change: Central Asia, East and Central Europe and Turkey*, Duke University Press.
- Deere, D., Murphy, K., Welch, F., 1995. "Employment and the 1990–1991 Minimum-wage Hike", *American Economic Review* pp.232–237.
- Dickens, R., S. Machin and A. Manning (1999). "The Effects of Minimum Wages on Employment: Theory and Evidence from Britain," *Journal of Labor Economics*, University of Chicago Press, vol. 17(1), pages 1-22.
- Eccles, M. and R. B. Freeman (1982) "What! Another Minimum Wage Study?," NBER Working Papers #0878
- Fraja, G. (de) (1999): "Minimum Wage Legislation, Productivity and Employment", *Economica*, 66 (264) 473-88, London.
- ILO(1992) General Survey of Reports on the Minimum Wage Fixing Machinery, International Labor Conference 79th Session, Geneva
- Krueger, A. B. and Summers, L. H, (1988 ) "Efficiency wages and the inter-industry wage structure", *Econometrica*, V 56, pp. 259-293.
- Machin, S. and A. Manning, (1994) "Minimum Wages, Wage Dispersion and Employment: Evidence from UK Wages Council", *Industrial and Labor Relations Review*, 47, pp.319-329
- Machin, S. and A. Manning, (1996) "Employment and the Introduction of the Minimum Wages in Britain", *Economic Journal* 106, pp.667-676



- McFadden D.L.,(1989),"A method of Simulated Moments for Estimation of Discrete Response Models without Numerical Integration", *Econometrica* 57, 995-1026.
- Mincer, J. (1985) "Intercountry Comparisons of Labor Force Trends and of Related Developments: An Overview" *Journal of Labor Economics*, 3(1), pp. 1-32.
- Moffitt, R. (1982) "The Tobit Model, Hours of Work and Institutional Constraints", *Review of Economics and Statistics*, Vol.64 No.3, pp.510-15.
- Moffitt, R. (1984) "The Estimation of a Joint Wage-Hours Labor Supply Model" *Journal of Labor Economics* Vol. 2, No. 4, pp. 550-566
- Neumark, D., W.Wascher (1995), "Minimum-wage effects on School and Work Transitions of Teenagers", *American Economic Review* pp.244–249
- OECD 2000 Country Report, OECD Webpage ([www.oecd.org](http://www.oecd.org))
- Oi, Walter (1962), "Labor as a Quasi-Fixed Factor", *Journal of Political Economy*, 70, pp. 538-555.
- Ozar, Ş.(1996), "*The Employment Aspects of the Informal Sector*" in Tuncer Bulutay (ed.), *Informal Sector (II)*, pp.175-203. Ankara: State Institute of Statistics
- Pencavel, J. "*Labor Supply of Men: A Survey*," in O.Ashenfelter and R. Layard (eds.), *Handbook of Labor Economics*, Vol. 1. North Holland, 1986.
- Rebitzer, J. and L.Taylor (1995), "The Consequences of Minimum Wage Laws: Some New Theoretical Ideas", *Journal of Public Economics*, February, pp.245-255
- Schultz, T. P. (1990) "Women's Changing Participation in the Labor Force: A World Perspective", *Economic Development and Cultural Change*, 38 (3), pp. 719-42.
- Shorter, Fred (1995), "The Crisis of Population Knowledge in Turkey." *New Perspectives on Turkey*, no.12 (Spring), pp.1-31.
- SIS (State Institute of Statistics) Webpage, HLFS Database
- Train, K., *Discrete Choice Methods with Simulation*, Cambridge University Press: Cambridge, 2003
- Tunali, I. (1997) "To Work or Not to Work: An Examination of Female Labor Force Participation Rates in Urban Turkey" *Proceedings of the 4th Annual ERF Conference held in Beirut*
- Tunali, I. and C. Baslevent (2005) "Female Labor Supply in Turkey" in S.Altug and A. Filiztekin(Eds). *The Turkish Economy: The Real Economy, Corporate Governance and Reform and Stabilization Policy*, Routledge Curzon Studies in Middle Eastern Economics
- TISK Yayinlari, [www.tisk.org.tr](http://www.tisk.org.tr)
- Wooldridge, Jeffrey, *Econometric Analysis of Cross Section and Panel Data*, MIT Press: Cambridge, 2001

## APPENDIXES

### I. UTILITY FUNCTION AND WORK DECISION

Following two graphs show the relationship between the work hours and the utility, keeping everything else constant for two different individuals. Both individuals have the same characteristics, except for the number of young kids. X-axis crosses the y-axis at  $U(h=0)$ , that is at the utility level from not working.

This first figure illustrates the utility function of an individual for whom not working is superior to working at any  $h$ . This individual is not going to work at  $h^*$ , since this local maximum implies a lower utility level than what he gets at  $h = 0$ .

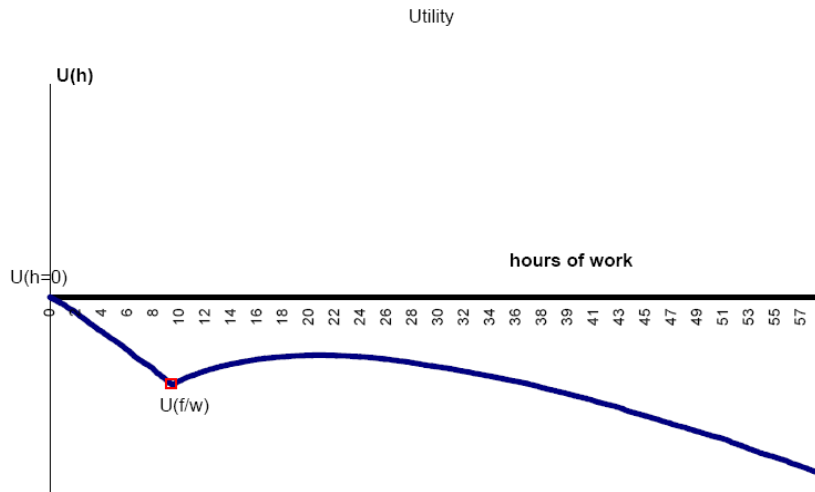


Figure A-1

The following individual has the same characteristics with the above individual except the number of young kids. As you can see the absolute required minimum is same for both individuals since only the productivity variables affect the location of this minimum. Unlike the above case, there is a positive  $h$  for this individual where her utility is maximized. She will work  $h^*$  (point C) if the required minimum hours is between points B and C. She will work her required minimum hours if the minimum is between C and D (note that for these points utility is higher than what it is at  $h = 0$ ). If the required minimum is more than D, she will not work, since now not working gives a higher utility compared to working at  $h^{min}$ .

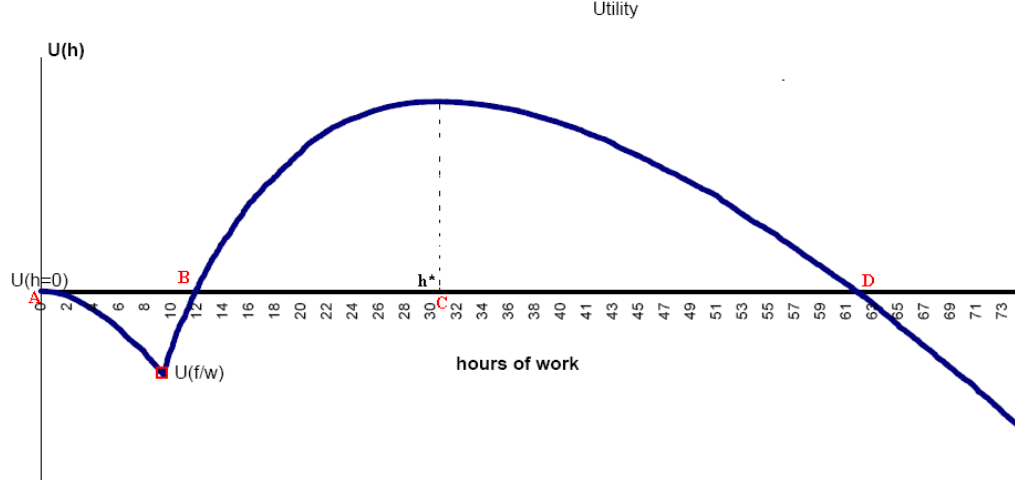


Figure A-2

## II. DERIVATION OF THE LIKELIHOOD FUNCTION

The individual's problem is to maximize

$$\begin{aligned}
 U &= U(C_i, L_i; A_i, \epsilon_i) \\
 &= \left( \frac{\alpha_2(T - L_i) - \alpha_1}{\alpha_2^2} \right) \exp \left( \frac{\alpha_2(\alpha_0 + \alpha_2 C_i + \alpha_3 A_i + \epsilon_{1i}) - \alpha_1}{\alpha_2 h_i - \alpha_1} \right)
 \end{aligned}$$

subject to the following set of constraints

$$\begin{aligned}
 C_i &\leq M_i + \gamma(\pi_i h_i - f) \\
 L_i + h_i &\leq T
 \end{aligned}$$

where  $A_i$  is a vector of demographic characteristics,  $M_i$  is non-labor income and  $C_i$  is the composite good (the numeraire),  $L_i$  is leisure, and  $T$  is the fixed weekly time endowment that can be divided between leisure and work.  $\gamma$  is a dummy which is equal to 1 if the individual works and 0 if not. Solution to this optimization problem gives

$$h_i^* = T - L_i^* = \alpha_0 + \alpha_1 \pi_i + \alpha_2(M_i - f) + \alpha_3 A_i + \epsilon_{i1}$$

as the desired hours equation. This model has two more latent indexes:

$$\pi_i = X_i \beta + \epsilon_{i2}$$

$$h_{ij}^{\min} = \frac{f}{\pi_i - w_j^{\min}}$$

Then for a worker

$h_i = h_i^*$  (*works desired hours*) if

$$h_i^* > h_{ij}^{\min} \text{ and } \pi_i > w_j^{\min}$$

$h_i = h_{ij}^{\min}$  (*works required minimum hours*) if

$$0 < h_i^* < h_{ij}^{\min} \text{ and } U(h_i = h_{ij}^{\min}) > U(h_i = 0)$$

$h_i = 0$  (*desires to work but is restricted*) if

$$0 < h_i^* < h_{ij}^{\min} \text{ and } U(h_i = h_{ij}^{\min}) < U(h_i = 0)$$

$h_i = 0$  (*does not want to work but is offered a job*) if

$$h_i^* \leq 0 \text{ and } \pi_i > w_j^{\min}$$

and

$h_i = 0$  (*can not work-no job is offered*) if

$$\pi_i < w_j^{\min}$$

Then the log-likelihood function is:

$$\log L = \sum_{h>0} \log Q + \sum_{h=0} \log q$$

where

$$Q = \left( \begin{array}{c} \left( \begin{array}{c} k(h | \text{Region I}, X_i, A_i, \sigma_1, \sigma_2, w_j^{\min}, M_i) \\ \Pr(\text{Region I} | X_i, A_i, \sigma_1, \sigma_2, w_j^{\min}, M_i) \end{array} \right) \\ + \\ \left( \begin{array}{c} k(h | \text{Region II}, X_i, A_i, \sigma_1, \sigma_2, w_j^{\min}, M_i) \\ \Pr(\text{Region II} | X_i, A_i, \sigma_1, \sigma_2, w_j^{\min}, M_i) \end{array} \right) \end{array} \right)$$

and

$$q = \begin{pmatrix} \Pr(\text{Region III} \mid X_i, A_i, \sigma_1, \sigma_2, w_j^{\min}, M_i) \\ + \Pr(\text{Region IV} \mid X_i, A_i, \sigma_1, \sigma_2, w_j^{\min}, M_i) \\ + \Pr(\text{Region V} \mid X_i, A_i, \sigma_1, \sigma_2, w_j^{\min}, M_i) \end{pmatrix}$$

( $k(\cdot)$  is the conditional probability density function of the hours of work variable given dependent variables, non-labor income and minimum wage levels and the unobserved preference and productivity shocks).

$$k(h \mid \text{Region I}, \cdot) = \frac{\Phi \left[ \frac{(h_i - f)X_i\beta - w_j^{\min}h_i}{\sqrt{(f - h_i)^2 \sigma_2^2}} \right] \phi \left[ \frac{h_i - \alpha_0 - (\alpha_1 - \alpha_2 f)X_i\beta - \alpha_2 M_i - \alpha_3 A_i}{\sqrt{(\alpha_1 - \alpha_2 f)^2 \sigma_2^2 + \sigma_1^2}} \right] \frac{1}{\sqrt{(\alpha_1 - \alpha_2 f)^2 \sigma_2^2 + \sigma_1^2}}}{\Pr(0 < h_{ij}^{\min} < h^*, U(h = h_{ij}^{\min}) > U(h = 0) \mid X_i, A_i, \sigma_1, \sigma_2, w_j^{\min}, M_i)}$$

$$k(h \mid \text{Region II}, \cdot) = \frac{[\Phi(Z_1) - \Phi(Z_2)] \left( \frac{w_j^{\min} f}{(h_i - f)^2 \sigma_2} \right) \phi \left( \frac{w_j^{\min} \frac{h_i}{h_i - f} - X_i\beta}{\sigma_2} \right)}{\Pr(0 < h_i^* < h_{ij}^{\min}, U(h_i = h_{ij}^{\min}) > U(h_i = 0) \mid X_i, A_i, \sigma_1, \sigma_2, w_j^{\min}, M_i)}$$

where

$$Z_1 = \frac{\log\left(\frac{\alpha_1 - \alpha_2 h_i}{\alpha_1}\right) [(\alpha_1 - \alpha_2 h_i)(\alpha_1)] - \alpha_2^2 \alpha_0 h_i - \alpha_2^3 h_i M_i - \alpha_2^2 \alpha_3 A_i h_i + \alpha_2 \alpha_1 h_i - \alpha_2^2 \alpha_1 w_j^{\min} h_i}{\alpha_2^2 h_i \sigma_1}$$

and

$$Z_1 = \frac{h_i - \alpha_0 - (\alpha_1 - \alpha_2 f)X_i\beta - \alpha_2 M_i - \alpha_3 A_i - (\alpha_1 - \alpha_2 f) \left[ w_j^{\min} \frac{h_i}{h_i - f} - X_i\beta \right]}{\sigma_1}$$

And

$$q = \begin{pmatrix} \Pr \left( \begin{aligned} & \left[ \left( \alpha_2 h_{ij}^{\min} - \alpha_1 \right) \exp \left( \frac{\alpha_2 (\alpha_0 + \alpha_2 M_i + \alpha_2 h_{ij}^{\min} w_j^{\min} + \alpha_3 A_i + \epsilon_{1i}) - \alpha_1}{\alpha_2 h_{ij}^{\min} - \alpha_1} \right) \right. \\ & \quad < \\ & \quad \left. (-\alpha_1) \exp \left( \frac{\alpha_2 (\alpha_0 + \alpha_2 M_i + \alpha_3 A_i + \epsilon_{1i}) - \alpha_1}{-\alpha_1} \right) \right] \\ & \quad , h_i^* > 0, \\ & \quad \pi > w_j^{\min} \mid X_i, A_i, \sigma_1, \sigma_2, w_j^{\min}, M_i \\ & + \Pr \left[ h_i^* < 0, \pi > w_j^{\min} \mid X_i, A_i, \sigma_1, \sigma_2, w_j^{\min}, m_i \right] \\ & + \Pr \left[ \pi < w_j^{\min} \mid X_i, A_i, \sigma_1, \sigma_2, w_j^{\min}, M_i \right] \end{aligned} \right) \end{pmatrix}$$

$$\begin{aligned}
&= \left( \Pr \left( \begin{aligned} &\left[ \left( \alpha_2 h_{ij}^{\min} - \alpha_1 \right) \exp \left( \frac{\alpha_2 (\alpha_0 + \alpha_2 m + \alpha_2 h_{ij}^{\min} w_j^{\min} + \alpha_3 A_i + \epsilon_{1i}) - \alpha_1}{\alpha_2 h_{ij}^{\min} - \alpha_1} \right) \right] \\ &\qquad < \\ &\left[ \begin{aligned} &(-\alpha_1) \exp \left( \frac{\alpha_2 (\alpha_0 + \alpha_2 M_i + \alpha_3 A_i + \epsilon_{1i}) - \alpha_1}{-\alpha_1} \right) \\ &\qquad , \quad X_i \hat{\beta} - w_j^{\min} > -\epsilon_{2i}, \\ &\alpha_0 + (\alpha_1 - \alpha_2 f) X_i \hat{\beta} + \alpha_2 M_i + \alpha_3 A_i > \epsilon_{1i} + \epsilon_{2i} (\alpha_1 - \alpha_2 f) \end{aligned} \right] \end{aligned} \right) \right) \\
&+ \Pr \left( \begin{aligned} &\left[ \begin{aligned} &\alpha_0 + \alpha_1 \left( X_i \hat{\beta} + \epsilon_{i2} \right)_i + \alpha_2 \left( M_i - f(X_i \hat{\beta} + \epsilon_{i2}) \right)_i \\ &\qquad + \alpha_3 A_i + \epsilon_{1i} < 0, \quad X_i \hat{\beta} + \epsilon_{i2} > w_j^{\min} \end{aligned} \right] \\ &\qquad + \Pr \left[ X \hat{\beta} + \epsilon_{i2} < w_j^{\min} \right] \end{aligned} \right) \right) \\
&= \left( \Pr \left( \begin{aligned} &\left[ \left( \alpha_2 h_{ij}^{\min} - \alpha_1 \right) \exp \left( \frac{\alpha_2 (\alpha_0 + \alpha_2 M_i + \alpha_2 h_{ij}^{\min} w_j^{\min} + \alpha_3 A_i + \epsilon_{1i}) - \alpha_1}{\alpha_2 h_{ij}^{\min} - \alpha_1} \right) \right] \\ &\qquad < \\ &\left[ \begin{aligned} &(-\alpha_1) \exp \left( \frac{\alpha_2 (\alpha_0 + \alpha_2 M_i + \alpha_3 A_i + \epsilon_{1i}) - \alpha_1}{-\alpha_1} \right) \\ &\qquad , \quad X_i \hat{\beta} - w_j^{\min} > -\epsilon_{2i}, \\ &\alpha_0 + (\alpha_1 - \alpha_2 f) X_i \hat{\beta} + \alpha_2 M_i + \alpha_3 A_i > \epsilon_{1i} + \epsilon_{2i} (\alpha_1 - \alpha_2 f) \end{aligned} \right] \end{aligned} \right) \right) \\
&+ \Phi \left[ \frac{-[\alpha_0 + (\alpha_1 - \alpha_2 f) X_i \hat{\beta} + \alpha_2 M_i + \alpha_3 A_i]}{\sqrt{\sigma_1^2 + (\alpha_1 - \alpha_2 f) \sigma_2^2}}, \frac{-X_i \hat{\beta} + w_j^{\min}}{\sigma_2} \right] + \Phi \left[ \frac{X_i \hat{\beta} - w_j^{\min}}{\sigma_2} \right] \right)
\end{aligned}$$